A STUDY OF NUMERICAL FORECASTING ERRORS

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ABSTRACT

Studies of numerical forecasting errors have revealed that the principal difficulties encountered in models currently operational at the Joint Numerical Weather Prediction Unit arise from errors in the 500-mb. forecasts. These errors are common to both barotropic and baroclinic models. The errors in the 1000–500-mb. thickness forecasts are considerably smaller. Results show that systematic errors are introduced (1) by the use of the geostrophic approximation and (2) by the approximations used on the boundaries. The cases presented in this paper show that the first type of error is virtually nonexistent in non-geostrophic, barotropic forecasts. Boundary errors are particularly serious in cases where the boundaries are meteorologically active, suggesting the future use of hemispheric forecast grids. The elimination of these two types of errors from all forecasts appears to be essential before the smaller errors, such as truncation errors and errors due to baroclinic development, non-adiabatic effects, etc., can be isolated and studied satisfactorily.

1. INTRODUCTION

In the normal routine of development work of the Joint Numerical Weather Prediction (JNWP) Unit a number of tests are made with a view toward isolating and removing certain types of forecasting errors. Some of the results of the tests are used immediately for model improvement. Other results, while yielding significant information, cannot be incorporated into operational forecasting for various reasons. The purpose of this paper is to record the results of several tests which would not otherwise be available for study elsewhere.

These tests which are being reported on by the authors have been suggested, conducted, and evaluated by a number of members of the JNWP Unit. Dr. Fred Shuman, Lt. Col. P. D. Thompson, USAF, Mr. L. Carstensen, Mr. G. Arnason, Lt. E. Carlstead, USN, Mr. C. Cave, and others have contributed to the work described below.

2. THE APPROXIMATION TO THE INITIAL WIND FIELD

Generally speaking, the most troublesome and significant errors found in numerical prediction arise directly from the difficulties in forecasting the 500-mb. flow. These difficulties are equally prominent in barotropic and multilevel forecasting. The first type of error to be discussed has been eliminated from the JNWP Unit barotropic forecasts, but is described below due to its importance. These errors are attributable to the use of the geostrophic approximation in describing the horizontal winds. They are manifested as (a) too strong deepening of Lows east of a strong current from the north, and (b) a tendency for erroneously large anticyclones to develop to the right of the jet stream (see Bolin [1]). The tendency for an erroneously large anticyclone to form is even further exaggerated if the jet stream is directed toward the north.

Errors introduced by the geostrophic approximation may be classified as arising from the fictitious divergence of the geostrophic wind, from the incorrect calculation of the vorticity when taking $\zeta_g=f^{-1}\nabla^2\phi$ while neglecting the term $f^{-1}\nabla\phi\cdot\nabla f$, and from the failure to take into account accelerations and divergence in the initial wind field. The symbols have their standard meaning, where f is the Coriolis parameter, ϕ is geopotential, and ζ_g is the geostrophic relative vorticity.

Several series of forecasts have been made with the use of different approximations to the wind. Such a series is presented in figures 1 to 7. The wind has been described by (a) the geostrophic wind, (b) a non-divergent, non-geostrophic wind, represented by the stream function ψ (see Shuman [2]), and (c) a wind which is non-accelerated and non-divergent (one could call this a "non-divergent geostrophic wind"), represented by a stream function S_2 (discussed by Shuman [2]) and defined by

$$\nabla^2 S_2=f^{-1}(\nabla^2\phi+\beta u_g)$$

where β is the northward variation of the Coriolis parameter, and u_g is the geostrophic west wind component.

These examples do not give typical, but extreme, results. They are presented to show the magnitude of errors that can be introduced by the geostrophic approximation. The situation presented was characterized by an unusually extensive meridional flow in the Gulf of Alaska and a strong zonal flow across the United States. The initial

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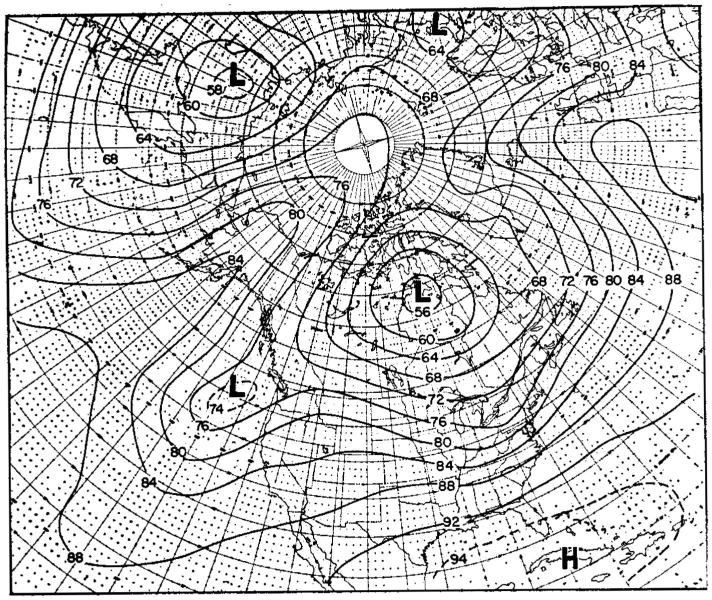


FIGURE 1.—Initial 500-mb. chart, January 10, 1957, 1500 GMT.

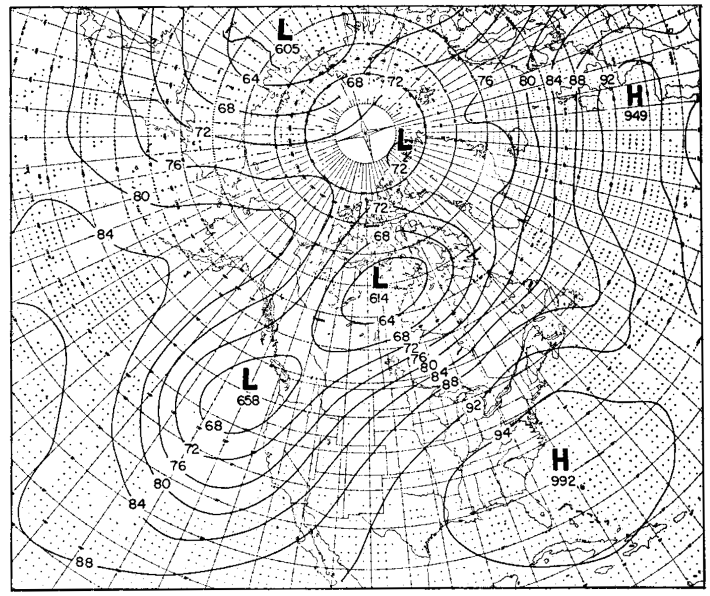


FIGURE 3.—36-hour geostrophic barotropic 500-mb. forecast from January 10, 1957, 1500 GMT.

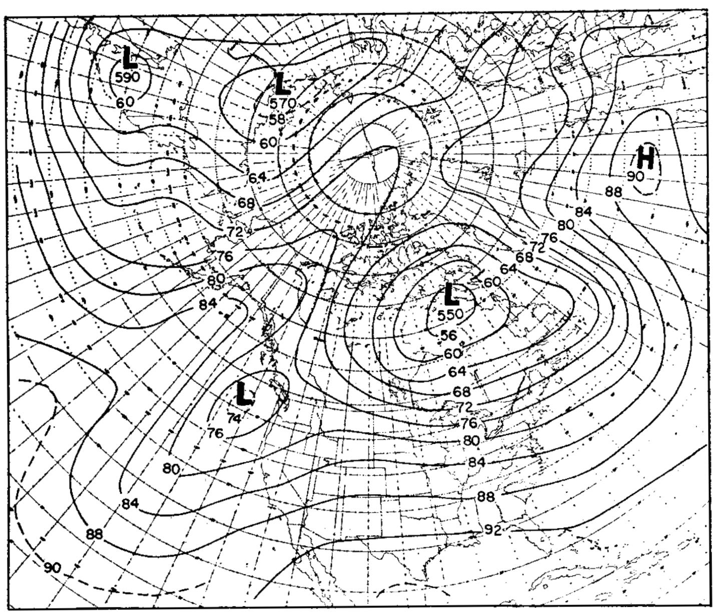


FIGURE 2.—Verifying 500-mb. chart, January 12, 1957, 0300 GMT

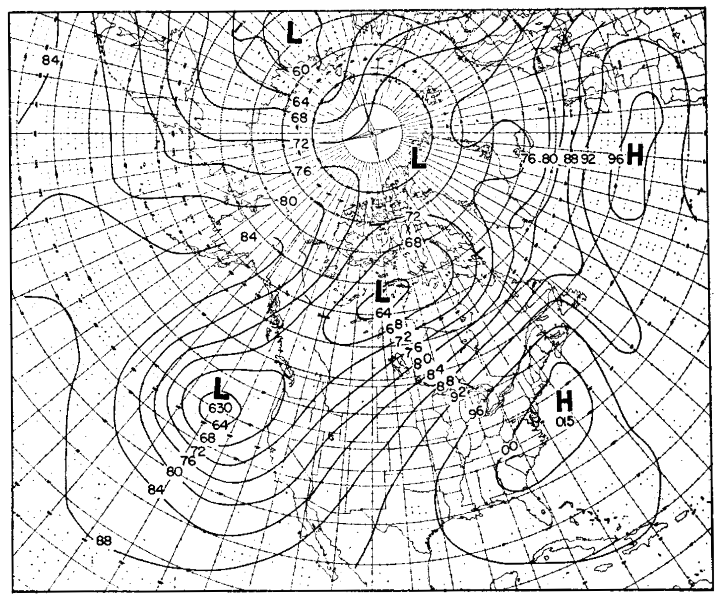


FIGURE 4.—36-hour geostrophic thermotropic 500-mb. forecast from January 10, 1957, 1500 GMT.

and verifying 500-mb. charts are shown in figures 1 and 2. The geostrophic barotropic forecast (with no terrain effects included) is shown in figure 3. In this forecast the excessive development of amplitude across North America has become ruinous. This difficulty is even more pronounced in the geostrophic thermotropic forecast shown in figure 4. For a detailed comparison of these two models see Thompson and Gates [3]. The differences between the two forecasts are that the equation for the thermotropic 500-mb. flow has an added Jacobian of the form $AJ(h,\nabla^2 h)$, h being the 500–1000-mb. thickness; there is also a term including terrain effects. The extra Jacobian has the effect of strengthening the flow in the forecast and has therefore accelerated the growth of the error. The mountain term is unimportant in comparison with the other errors in this case.

Figures 5 and 6 show the barotropic forecasts made with S_2 and ψ, respectively. Each of these is greatly improved over the geostrophic forecasts. Note that the S_2 forecast has an erroneous 19,600-ft. contour in the southeastern States, which does not appear in the ψ forecast. Finally, the thermotropic forecast made with S_2 winds at both the 1000 and 500-mb. levels is shown in figure 7. Further tests (not shown) have been made where the βu_g term was deleted from the S_2 stream function. These indicated that the additional contribution of this term is relatively unimportant, resulting mainly in a slightly stronger belt of westerlies.

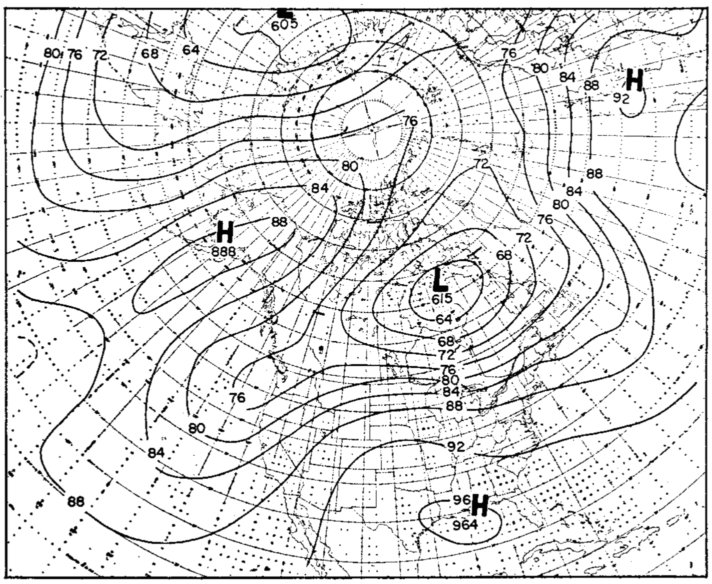


FIGURE 5.—36-hour barotropic 500-mb. forecast made using S_2 stream function from January 10, 1957, 1500 GMT.

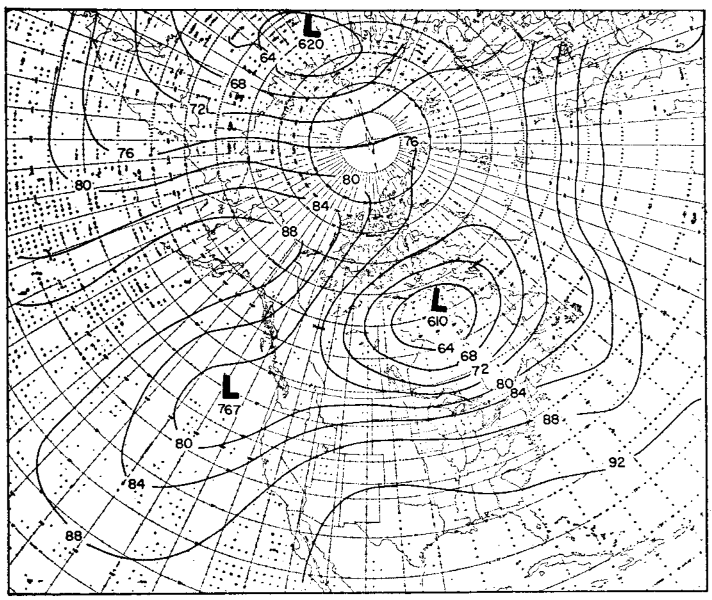


FIGURE 6.—36-hour barotropic 500-mb. forecast made using ψ stream function from January 10, 1957, 1500 GMT.

A comparison of figures 3 and 5 indicates that most of the error found in the geostrophic barotropic forecast was removed by use of the S_2 wind approximation. This leads to the conclusion that the divergence of the geostrophic wind was the largest contributing factor to the large errors of figures 3 and 4, in agreement with Shuman [2]. The further improvement exhibited by the ψ forecast is shown in figure 6. One may conclude that a non-geostrophic wind should be used in numerical prediction, since the errors introduced by the geostrophic approximation can be of such a large magnitude as to obscure all other types of error.

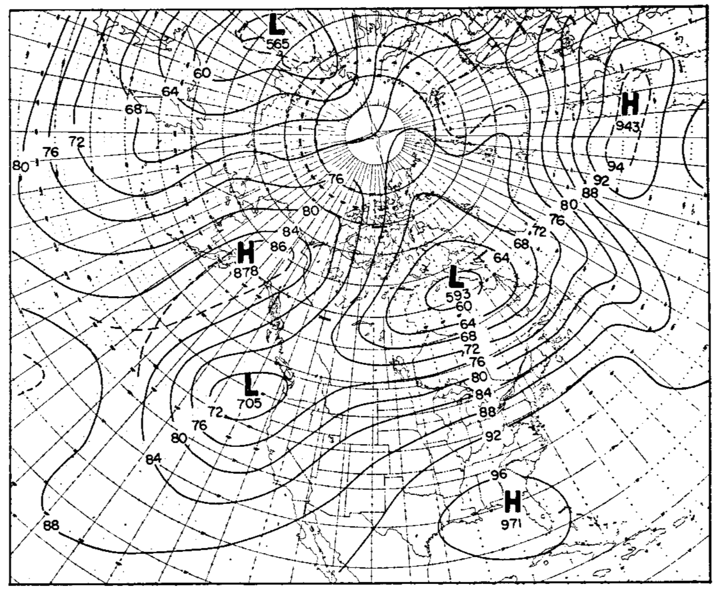


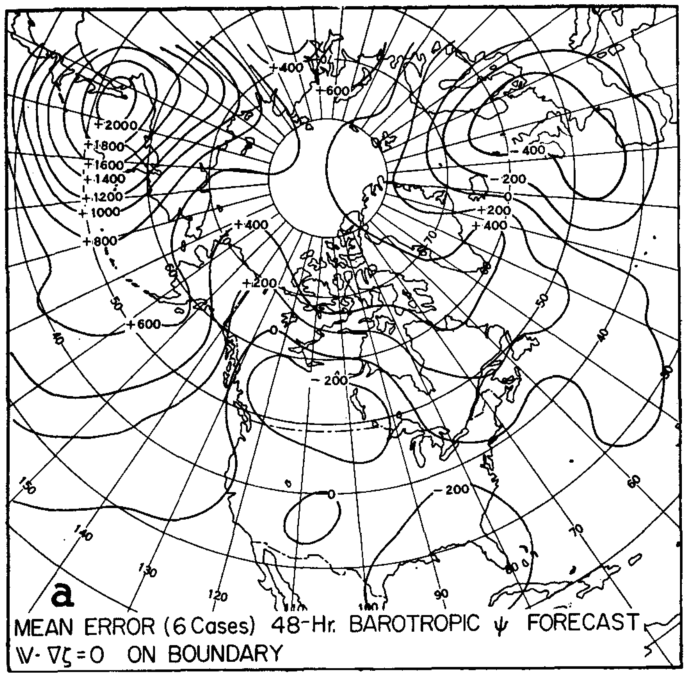
FIGURE 7.—36-hour thermotropic 500-mb. forecast made using S_2 stream function at 500 and 1000 mb. from January 10, 1957, 1500 GMT.

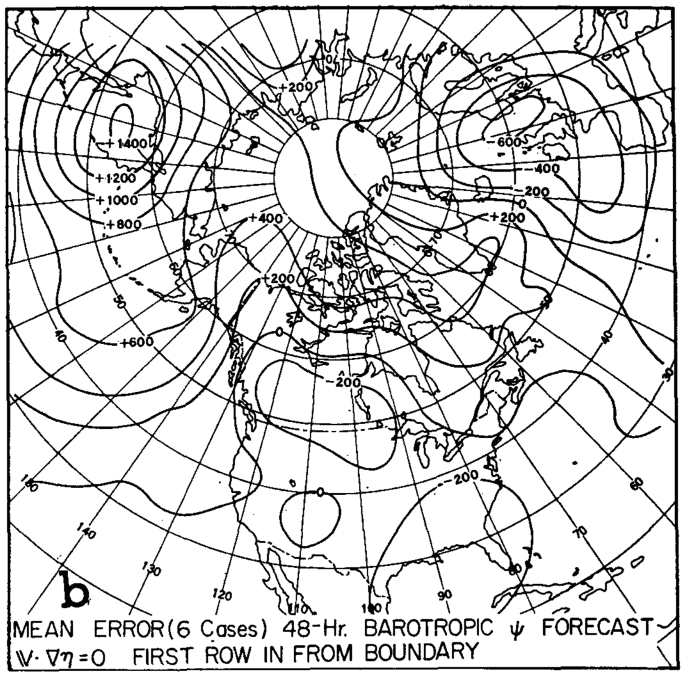
3. BOUNDARY CONDITIONS AND RELATED ERRORS

Another important source of error in numerical prediction is the condition imposed on the lateral boundaries. It should be relatively easy to specify a set of realistic and accurate enough boundary conditions if the boundaries fall in areas which are meteorologically inactive; i. e., with persistent, stagnant circulations. Such an arrangement does not seem to be possible short of using a hemispheric grid. The grid used by the JNWP Unit in 1956 and the first half of 1957 has a Pacific boundary which is meteorologically very active, with variable areas of strong inflow and strong change. It is our experience that the predictions for areas inside boundaries where strong inflow occurs are usually characterized by strong erroneous height rise (the stronger the flow, the greater the rise) particularly if the flow has a southerly component. Smaller erroneous falls are found near outflow boundaries.

A set of six situations from December 1956 and January 1957 has been used for testing remedies for boundary errors. Although the meteorological situations varied among these cases, the large-scale fields of error were relatively similar. The forecasts were made with a stream function ψ given by the balance equation [2] in a barotropic prediction model. Boundary errors are introduced by the boundary conditions in the balance equation, boundary conditions in the forecast model, and boundary problems introduced by smoothing. The boundary conditions used in solutions of the balance equation are essentially that the flow component normal to the boundaries is set equal to the corresponding component of the geostrophic flow.

The boundary conditions previously used in the operational barotropic forecast were (a) constant values of





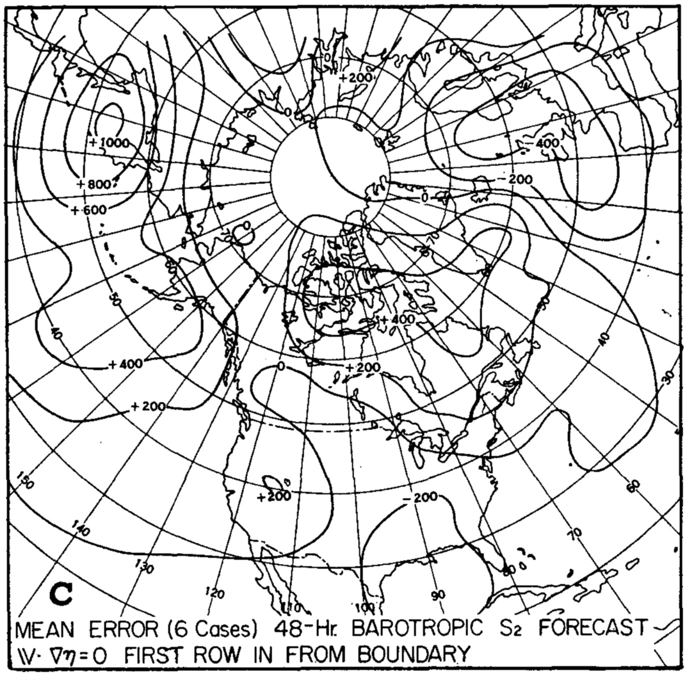


FIGURE 8.—Mean algebraic errors of 48-hour barotropic forecasts made from six initial maps. Errors in feet.

stream function on the boundary points through the forecast period, and (b) zero relative vorticity at the boundaries. The average algebraic errors of the 48-hour forecasts for the six cases mentioned above are shown in figure 8a. Of special interest are the 2000-ft. positive errors over the Sea of Okhotsk and the 400-ft. negative errors over the North Sea.

The forecast program was then changed so that the initial values of vorticity on the rows and columns adjacent to the boundaries were retained with no change during the forecast (zero Jacobians at these points). The average algebraic error for the same six cases is shown in figure 8b. Although the error was increased slightly over the North Sea, it was decreased by 600 ft. over the Sea of Okhotsk (inflow boundary). It is suggested that the reason for this improvement was that the new boundary conditions permitted a positive vorticity transport into the grid from areas where troughs were found at the initial time. It is well known that the East Asia trough is very persistent.

A further reduction in the average algebraic error in this area was found when S_2 was used as the stream function, as shown in figure 8c. This suggests that the boundary conditions in the more complete form of the balance equation may be responsible for some of the difficulty. One should not infer from these results that S_2 is a better stream function for barotropic forecasting than ψ, since only algebraic boundary errors have been discussed here.

Another experiment in the study of boundary errors was performed by making a set of non-geostropic barotropic forecasts with the grid located in two different positions on each map on each of three different days. The first of the two positions, referred to as "normal", is the same as that shown in figure 8. The second grid position, referred to as "rotated", was obtained by rotating the first one 180 degrees about the north pole. The error of each forecast was then averaged along each longitude from 35° N. to 90° N. The errors for each of the

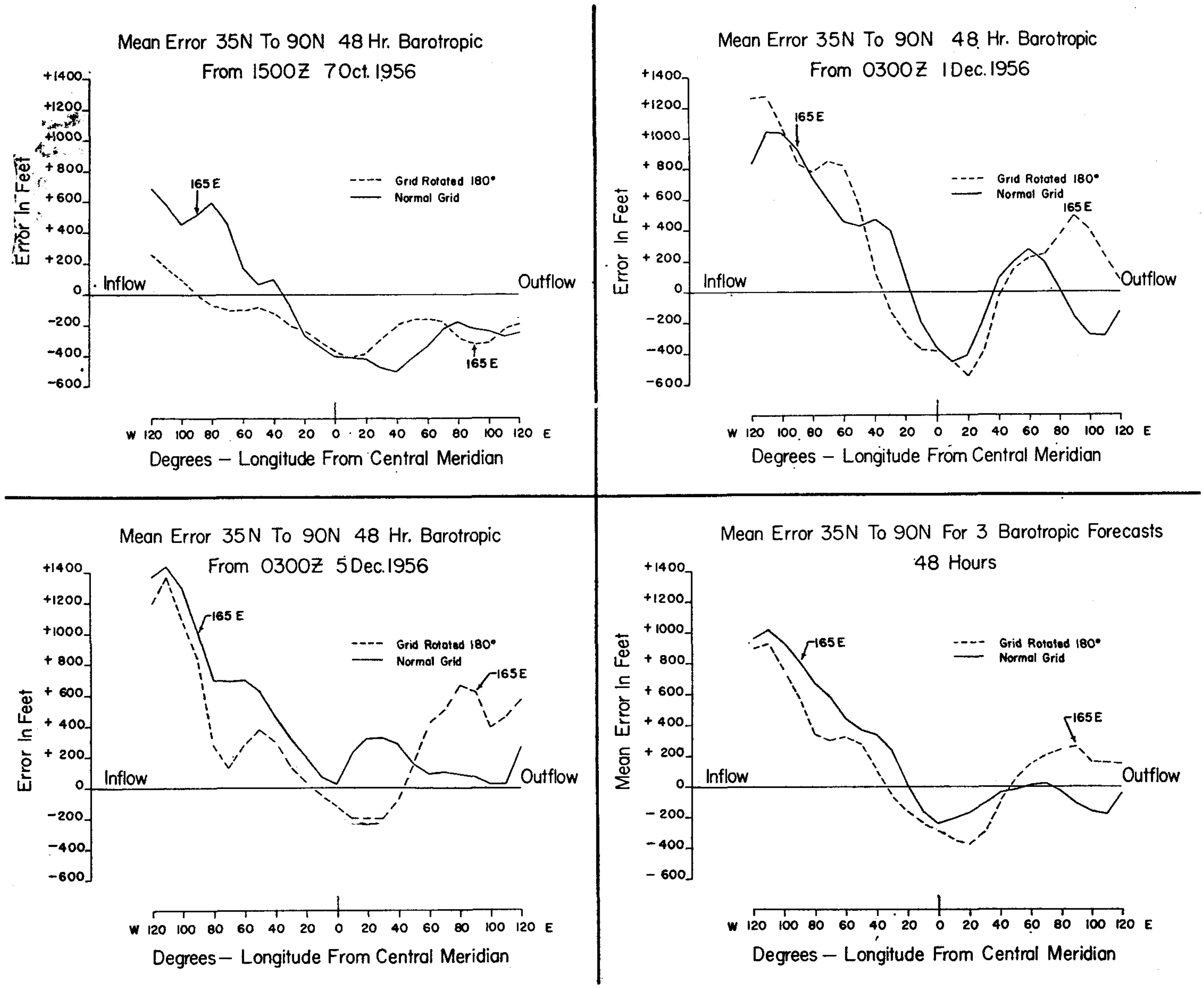


FIGURE 9.—Mean algebraic errors for three sets of non-geostrophic barotropic forecasts as a function of longitude in the grid. Errors in feet.

three days as well as the mean for all three are plotted against longitude in the grid in figure 9. In looking at this figure one gets the distinct impression that the largest forecast errors are a function of position in the grid and move with the grid as it is rotated.

This is not true of all significant errors, however, as can be seen by inspection of the points labeled "165° E." There is a tendency for a significant positive error to occur at or west of this longitude regardless of the grid position. A suggested source for this error is the effect of the divergence induced by the Asian Plateau. This barotropic model ordinarily takes into account the large-scale mountain effect. It was, however, not feasible to do so in this set of eight forecasts. The next barotropic model to be used by the JNWP Unit will be computed on a hemispheric grid, with which it should be possible to include the large-scale effect of the Asian Plateau.

The large-scale errors which are a function of position in the grid can arise from erroneous boundary conditions and truncation errors, the latter resulting from directional properties of the finite-difference Jacobian operators. In a further study of contributing factors to these errors, a fictitious geopotential field yielding 25-knot zonal winds throughout the entire grid was "balanced" to obtain the corresponding ψ field. This field was then used as input data for a 48-hour barotropic forecast. The results of this experiment are shown in figure 10. At least part of the observed error appears to be caused by the boundary conditions used in solving the balance equation. The initially circular ϕ field is distorted slightly, and the resultant long wave (with an amplitude of only 20 feet) in the ψ field grows with time.

When the hemispheric grid is put into operation, the boundary assumptions used in solving the balance equa-

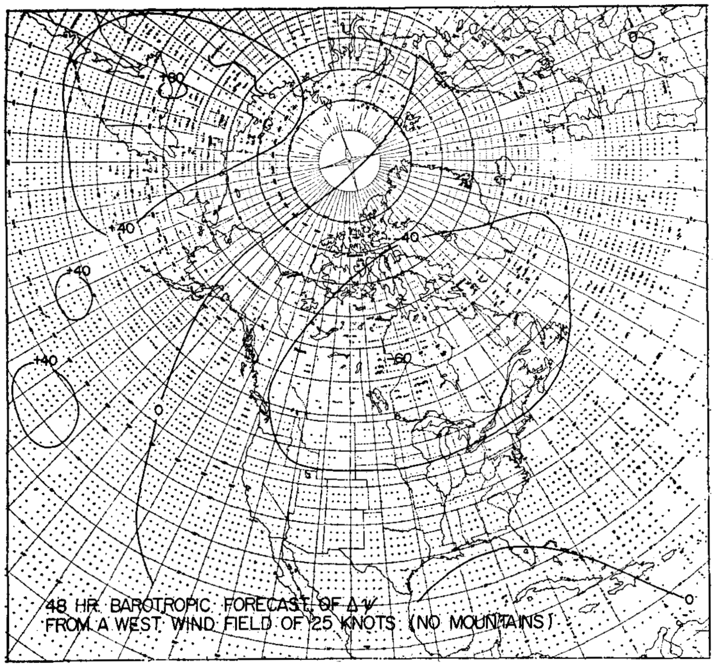


FIGURE 10.—48-hour barotropic change in stream function ψ (scaled as a 500-mb. height) from a 25-knot, zonal wind field. Change in feet. No mountains.

tion and those used in the forecast models themselves will be more realistic. Boundary errors should become insignificant. It should then be possible to isolate and study the smaller errors which are now obscured by larger ones.

4. TWO-LEVEL FORECASTS

Numerous references appear in the literature to the process of baroclinic development in which circulation is created or destroyed on a large scale at 500 mb. (see Charney [4]). Without disputing the fact that such developments do occur, one must remark that they are quite rare. A chronological series of charts of geostrophic absolute vorticity at 500 mb. shows many apparent circulation increases as cyclonic vorticity associated with shear becomes associated with curvature. Most of these apparent increases are erroneously indicated by the geostrophic wind. At the JNWP Unit an extended series of 500-mb. absolute vorticity charts of the non-divergent, non-geostrophic wind has been prepared. These show a remarkable preservation of the size and intensity of the 500-mb. circulation centers from day to day (Vederman and Hubert [5]). In fact, most of the *apparent* circulation changes occur in areas of sparse data. One can conclude that although baroclinic development may at times present a severe local problem in barotropic forecasting, it does not account for a very large fraction of the errors currently encountered in numerical prediction.

A quasi-geostrophic two-parameter model similar to the "thermotropic" model described by Thompson and Gates [3] has been used by the JNWP Unit to obtain daily 1000-mb. forecasts. The details in which this model differs from that of Thompson and Gates are: (a) the JNWP model includes the effect of large-scale topographic features, and (b) in the JNWP model the absolute vorticity is not replaced by a constant, standard value when it is used as a coefficient—it varies freely.

The errors in the 500-mb. forecasts produced by this model appear to arise mainly from the use of the geostrophic approximation and from boundary errors. The errors in the thickness forecasts are difficult to diagnose, since the 500-mb. flow (including its errors) is used in computing the thickness changes. Some light is shed on this problem by the charts of figures 11–14. Figures 11 and 12 show the mean algebraic and absolute errors respectively in the 500-mb. forecasts for a half-month period. From the close similarity of these charts it can be concluded that the 500-mb. errors were mostly systematic during this period. Figures 13 and 14 show the mean algebraic and absolute errors of the thickness forecasts. These are significantly smaller than the corresponding 500-mb. errors. They also appear to be systematic in the Pacific, eastern Canada, and the Atlantic. Furthermore, if one compares the systematic errors in the 500-mb. forecast (fig. 11) with those in the thickness forecast (fig. 13), it appears that much of the thickness error is a result of the larger 500-mb. error. For example, the negative thickness error south of Greenland is in the same area as an erroneous 500-mb. flow from the colder area of the map, while the positive thickness error over southeastern Canada is in the same area as an erroneous 500-mb. flow from the south.

Some of the thickness error arises from the use of a constant mean value for vertical stability across the whole map. Maps of stability between 1000 mb. and 500 mb. have been produced for individual days, showing on a typical day lateral variations in stability such that the highest stability was 240 percent of the lowest. Several forecasts have been made with different (3-level and thermotropic) models using varying values for the static stability. The results can be summarized by saying that the models responded to stability changes in the same way as the atmosphere does; i. e., with high stability the coupling between levels was very loose, and vice versa. One forecast was made in which, by accident, the vertical stability was set at an extremely large value. In this case, the changes in thickness and 500-mb. height were apparently unrelated.

The 1000-mb. forecasts from the thermotropic model are obtained by subtracting the 1000–500-mb. thickness forecast from the 500-mb. forecast. It appears, then, that the most effective way of improving the thickness (and 1000-mb.) forecast is to improve the 500-mb. forecast. It is also concluded that lateral variations of vertical stability must eventually be considered.

Inclusion of the large-scale effect of mountains is important to all forecasts, especially those of the lower levels. These will not be discussed in any detail except

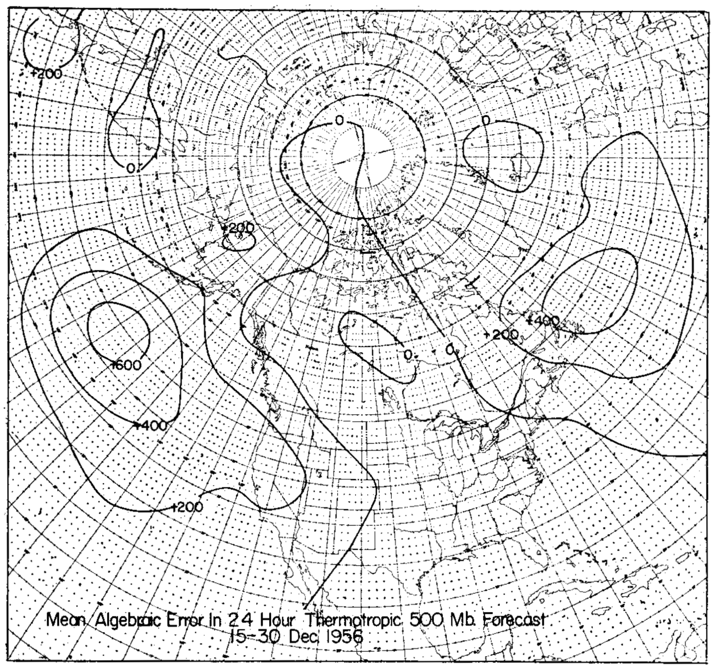


FIGURE 11.—Mean algebraic error of thermotropic 500-mb. forecasts, December 15–30, 1956. Errors in feet.

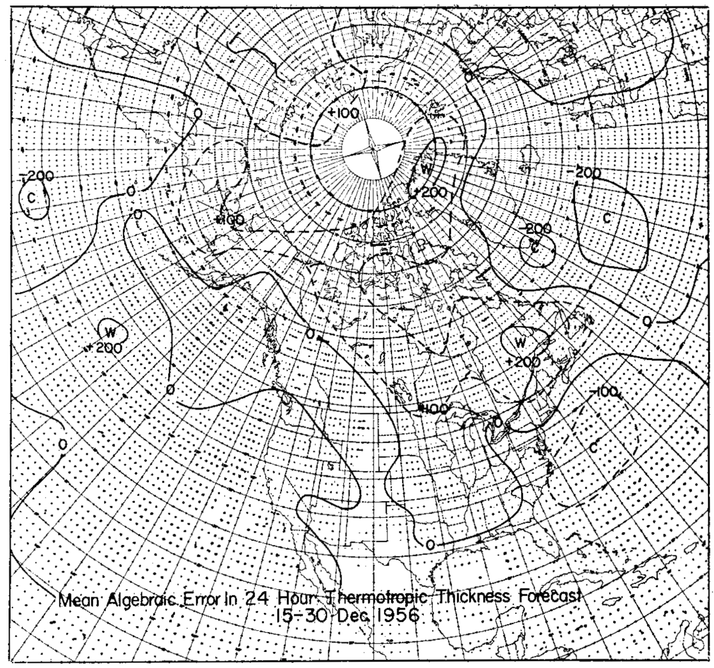


FIGURE 13.—Mean algebraic error of the thickness (500–1000 mb.), December 15–30, 1956. Errors in feet.

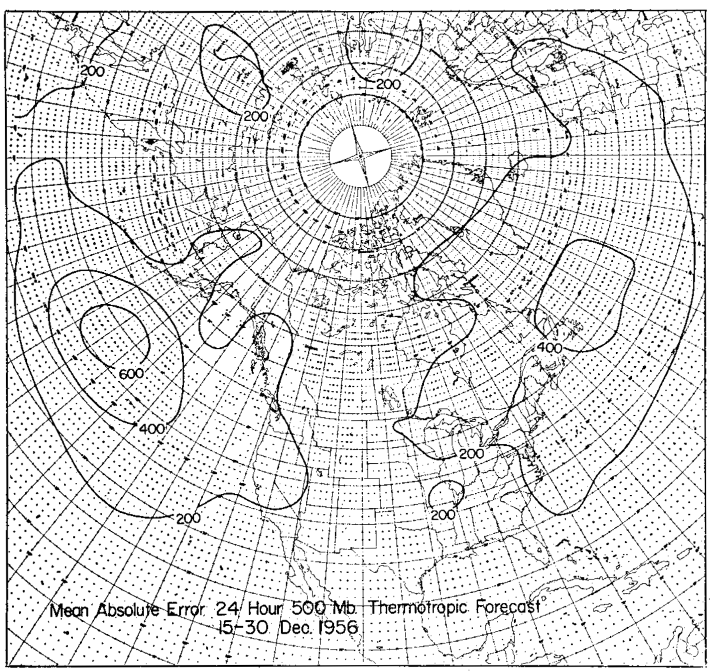


FIGURE 12.—Mean absolute error of thermotropic 500-mb. forecasts, December 15–30, 1956. Errors in feet.

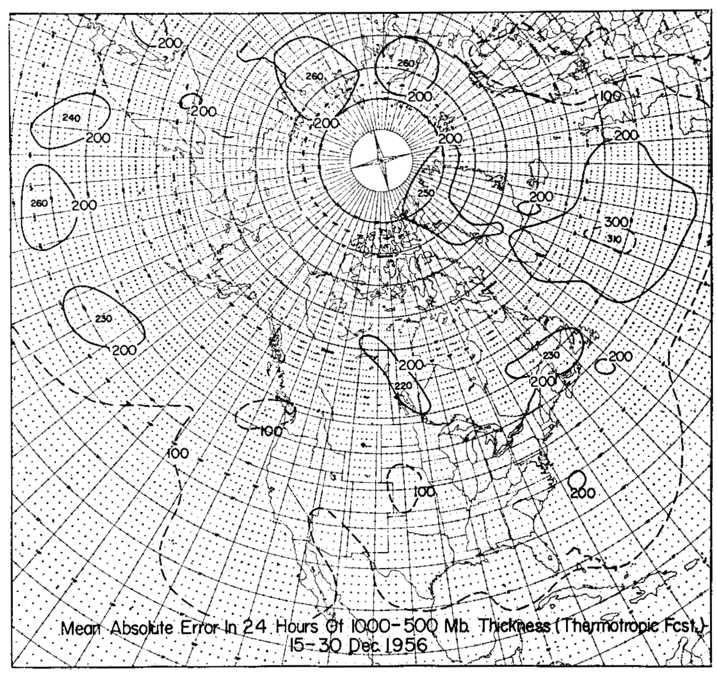


FIGURE 14.—Mean absolute error of the thickness (500–1000 mb.), December 15–30, 1956. Errors in feet.

to say that if f is used instead of $\zeta + f$ for the coefficient of the divergence in the vorticity equation, the computation of mountain effects is not nearly so successful. Smoothing of the terrain does not seem to be especially important unless it is done excessively; e. g., enough to double the apparent lateral extent of Greenland. This leads to undersirable results. Inclusion of mountain effects in the forecasts has a quite spectacular result over Greenland, where it prevents cyclones from crossing the high terrain, steering them around the coasts. Cyclone development on the lee side is clearly encouraged by the mountain effects in the model.

In order to see clearly the results of including a mountain effect in the barotropic model, the experiment with the 25-knot, zonal wind (fig. 10) was repeated with terrain included. The results are shown in figure 15. In the barotropic model, a fixed fraction of the 500-mb. wind is assumed to apply at the surface of the ground.

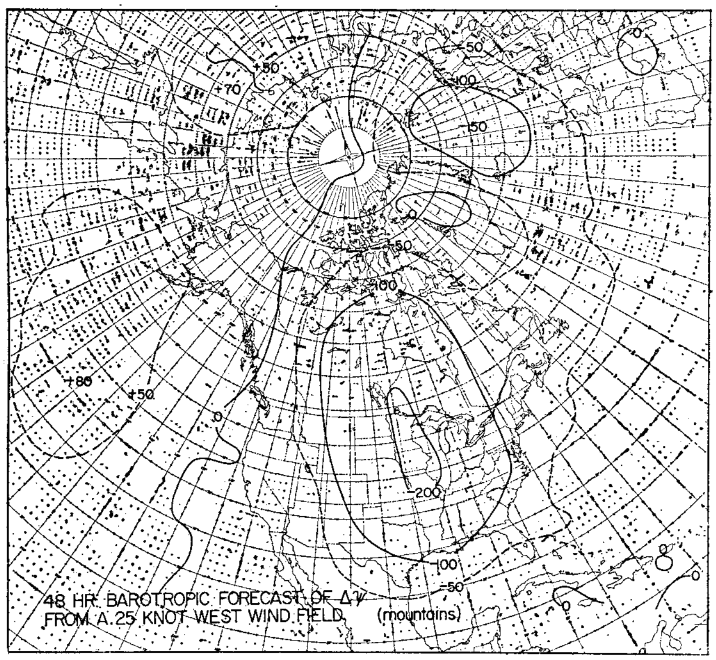


FIGURE 15.—48-hour barotropic change in stream function ψ (scaled as 500-mb. height) from a 25-knot, zonal wind field. Change in feet. Mountains included.

In spite of the crudeness of this approximation, the location and magnitude of the mountain waves appear to be quite realistic. Subtracting the values in figure 10 from those in figure 15 should eliminate the boundary errors of this experiment, leaving only the effect of the mountains.

5. CONCLUSIONS

In conclusion, the most serious errors in the models currently used in numerical prediction arise from difficulties encountered in forecasting the 500-mb. flow. The most important sources of error are the geostrophic approximation and the boundary conditions. The geostrophic approximation has been removed from the barotropic forecasts and must be removed from the more complicated models as well, since in extreme cases it can be responsible for very large forecast errors. No really satisfactory boundary conditions have been found for use along boundaries which are meteorologically active; i. e., characterized by strong flow or by significant changes in the flow pattern. It seems that the only workable solution is to move the boundaries of the forecast area into the low latitudes where the 500-mb. flow patterns are relatively weak and stagnant.

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